Pergamon

Int. J. Heat Mass Transfer. Vol. 40, No. 7, pp. 1551–1557, 1997

0017–9310/97 \$17.00+0.00

PII: S0017–9310(96)00180–9

Time-dependent free convection motion and heat transfer in an infinite porous medium induced by a heated sphere

R. GANAPATHY
Department of Mathematics, National College, Tiruchirapalli 620001, India

(*Received for publication* 18 *June* 1996)

Abstract—Buoyancy driven thermal convection due to the presence of a sphere of constant heat flux in an unbounded fluid-saturated porous medium is studied analytically. Transient and steady-state solutions have been obtained for the velocity and temperature fields in the form of series expansions in the Rayleigh number, which is based on the permeability of the porous medium and the heat flux from the sphere. All discussions are based on the assumptions that the flow is governed by Darcy's law and that the thermal Rayleigh number is small. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

Free convection flow and heat transfer due to the presence of a heated sphere embedded in an unbounded porous medium is one of the contemporary subjects of study owing to its wide-ranging applications in a number of fields such as chemical engineering, thermal insulation systems, and nuclear waste management. Ever since the work of Yamamoto [1], there has been a spate of research papers on this topic. More recently Lai and Kulacki [2] made an extension of Yamamoto's work to determine the impact of species concentration gradients upon the thermally driven flow. With the possible exception of the work of Ganapathy and Purushothaman [3], wherein the authors considered the impact of periodic variations in the temperature of the sphere on the essential flow characteristics, most of the existing studies were concerned only with steady-state solutions with constant temperature spheres. However, quite often the heated sphere is buried instantaneously and as a result, a knowledge of the transient behaviour of the flow and heat transfer becomes essential. Furthermore, in a practical situation such as the cooling of the components of electrical and electronic equipment and the management of nuclear waste, the appropriate boundary condition is that the total heat flux on the surface of the sphere is specified rather than that of the total prescribed wall temperature. It is therefore worthwhile to present a solution to this fundamental problem of penetrative convection due to a sphere of constant heat flux, buried instantaneously in an infinite porous medium and to investigate the ensuing flow field and heat transfer in the context of thermal flows in porous media.

The present work relies on the asymptotic expansions in the Rayleigh number to obtain the solutions of the temperature and flow fields in the system and hence the results reported here are representative of the low Rayleigh number regime. The sphere is assumed to be of constant heat flux, the fluid saturating the medium to have Boussinesq incompressibility, and the fluid flow to be governed by Darcy's law.

The mathematical problem is formulated in Section 2, the transient state is discussed in Section 3 and the steady-state in Section 4. Finally we conclude the study with a review of the results obtained.

2. MATHEMATICAL FORMULATION

We consider the natural convection around a sphere of radius a and of constant heat flux Q, buried instantaneously in an unbounded fluid-saturated porous medium of low permeability. The medium is assumed to be rigid, homogeneous and isotropic and the fluid saturating the medium has Boussinesq incompressibility with the density–temperature relation

$$\rho = \rho_\infty[1-\beta(T-T_\infty)] \tag{1}$$

where ρ is the fluid density, T is the temperature and β is the volumetric coefficient of thermal expansion and the subscript ∞ denotes a reference state.

A spherical-polar coordinate system (r, ϕ, θ) is chosen (Fig. 1), with the origin at the centre of the sphere and the axis $\phi = 0$ vertically upwards. Since the vertical axis is parallel to the gravity vector, the problem is symmetrical in the angular direction θ and consequently, neither θ nor the θ-component of velocity appears in the analysis. According to the Darcy flow model [4], the equations describing the conservation of mass, momentum and energy in the medium in the absence of dispersion effects are:

NOMENCLATURE

a	radius of the sphere [m]
c_p	specific heat of fluid at constant pressure [$m^2\ s^{-2}\ K^{-1}$]
e_i	exponential integral, equation (22c)
f	function of η, equation (19)
$f_{1,2,3}$	functions of η, equation (22a)
G	function of η, equation (24)
g	gravitational acceleration [ms^{-2}]
K	premeability of the porous medium [m^2]
k	thermal conductivity [$kg\ m\ s^{-3}\ K^{-1}$]
P	fluid pressure [$kg\ m^{-1}\ s^{-2}$]
Q	heat flux [$kg\ s^{-3}$]
R	non-dimensional radial coordinate
R_0	non-dimensional radius of the sphere ($a/\sqrt{K}$)
Ra	thermal Rayleigh number, ($\beta g K^2/\alpha\nu k)Q$
r	radial coordinate [m]
s	surface of the unit sphere
T	temperature [K]
t	time [s]
U	non-dimensional radial velocity
u	radial velocity [$m\ s^{-1}$]
V	non-dimensional transverse velocity
v	transverse velocity [$m\ s^{-1}$]
X	function of η_0, equation (22c).

Greek symbols

α	effective thermal diffusivity of the porous medium [$m^2\ s^{-1}$]
β	coefficient of thermal expansion [K^{-1}]
λ	porosity of the porous matrix
θ	azimuthal angle
ϕ	meridian angle
η	similarity variable, $R/2\sqrt{t}$
η_0	similarity variable, $R_0/2\sqrt{t}$
μ	fluid viscosity [$kg\ m^{-1}\ s^{-1}$]
ν	kinematic viscosity [$m^2\ s^{-1}$]
ε	dummy integration variable, equation (22c)
ρ	fluid density [$kg\ m^{-3}$]
σ	heat capacity ratio, equation (6)
ψ	stream function [$m^3\ s^{-1}$].

Subscripts

0	zero-order solution
1	first-order solution
2	second-order solution
∞	reference state
f	fluid phase
s	solid phase.

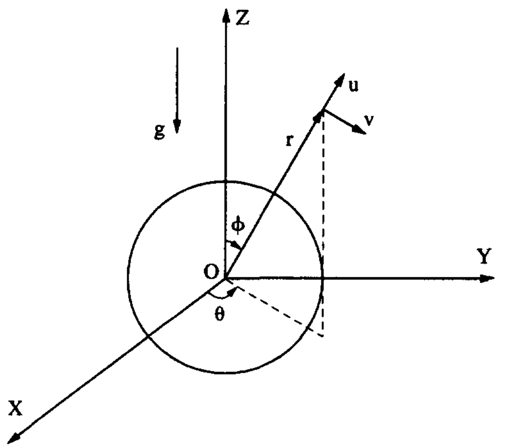

Fig. 1. Configuration of interest. Spherical-polar coordinate system (r, ϕ, θ) with the origin of the system at the centre of the sphere and the $\phi = 0$-axis parallel to the gravity vector.

$$\frac{\partial}{\partial r}(r^2 u \sin\phi) + \frac{\partial}{\partial \phi}(rv\sin\phi) = 0 \quad (2)$$

$$u = -\frac{K}{\mu}\left(\frac{\partial P}{\partial r} + \rho g \cos\phi\right) \quad (3)$$

$$v = -\frac{K}{\mu}\left(\frac{1}{r}\frac{\partial P}{\partial \phi} - \rho g \sin\phi\right) \quad (4)$$

$$\sigma\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial r} + \frac{v\partial T}{r\partial\phi} = \frac{\alpha}{r^2}\left[\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial T}{\partial\phi}\right)\right] \quad (5)$$

where u and v are the radial and transverse components of the velocity, K is the medium permeability, P is the pressure, μ is the coefficient of viscosity, g is the gravitational acceleration, α is the effective thermal diffusivity and σ is the heat capacity ratio given by

$$\sigma = \lambda + (1-\lambda)(\rho c_p)_s/(\rho c_p)_f \quad (6)$$

where λ is the porosity of the porous matrix, c_p is the specific heat of the fluid at constant pressure and the subscripts 'f' and 's' refer to the fluid and solid phases, respectively. In writing the above equations we have assumed that both the medium and the saturating fluid are in thermal equilibrium and that all the physical quantities are constant except in the buoyancy term.

We take advantage of the continuity equation (2) to define a stream function ψ such that

$$u = (r^2 \sin\phi)^{-1}\,\partial\psi/\partial\phi, \quad v = -(r\sin\phi)^{-1}\,\partial\psi/\partial r \quad (7)$$

and eliminate the pressure terms in equations (3) and

(4) by cross-differentiation. Introducing the non-dimensional quantities

$$R = r/\sqrt{K}, \quad t_* = \alpha t/K\sigma,$$
$$\psi_* = \psi/\alpha\sqrt{K}, \quad T_* = (T - T_\infty)/(Q/k)\sqrt{K} \quad (8)$$

where k is the thermal conductivity, we finally obtain for the conservation of momentum and energy in the non-dimensional form (after dropping the asterisk)

$$\frac{1}{R^2}\frac{\partial}{\partial\phi}\left(\frac{1}{\sin\phi}\frac{\partial\psi}{\partial\phi}\right) + \frac{1}{\sin\phi}\frac{\partial^2\psi}{\partial R^2} = Ra\left(\cos\phi\frac{\partial T}{\partial\phi} + R\sin\phi\frac{\partial T}{\partial R}\right) \quad (9)$$

$$\frac{\partial T}{\partial t} + \frac{1}{R^2\sin\phi}\frac{\partial(\psi, T)}{\partial(\phi, R)} = \frac{1}{R^2}\left[\frac{\partial}{\partial R}\left(R^2\frac{\partial T}{\partial R}\right) + \frac{1}{\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial T}{\partial R}\right)\right] \quad (10)$$

where

$$\frac{\partial(\psi, T)}{\partial(\phi, R)} = \frac{\partial\psi}{\partial\phi}\frac{\partial T}{\partial R} - \frac{\partial\psi}{\partial R}\frac{\partial T}{\partial\phi} \quad (11)$$

and

$$Ra = (\beta g K^2/\alpha\nu k)Q, \quad (12)$$

is the thermal Rayleigh number, based on the permeability of the medium and the specific heat flux at the surface of the sphere. The non-dimensional form of the velocity components are given by

$$(U, V) = (\sqrt{K}/\alpha)(u, v). \quad (13)$$

Accordingly, the initial and boundary conditions reduce to

$$U = V = T = 0 \quad \text{at } t = 0,$$
$$U, V, T \to 0 \quad \text{as } R \to \infty,$$
$$\partial U/\partial\phi = V = \partial T/\partial\phi = 0 \quad \text{at } \phi = 0, \pi,$$
$$(R^2\sin\phi)^{-1}\,\partial\psi/\partial\phi = 0,$$
$$\partial T/\partial R = -1 \quad \text{on } R = R_0, \quad (14)$$

where R_0 $(= a/\sqrt{K})$ is the non-dimensional radius of the sphere.

3. TRANSIENT STATE

Consistent with the hypothesis that the thermal Rayleigh number is small, we seek a perturbation solution by assuming power series expansions for ψ and T in the form:

$$(\psi, T) = \sum_0^\infty Ra^n(\psi_n, T_n) \quad (15)$$

with similar expressions for U and V to satisfy the conditions of equation (14). We substitute equation (15) into equations (9) and (10) and equate terms of equal powers in Ra. The appropriate boundary conditions are obtained from equation (14) with the help of equation (15).

As ψ_0 corresponds to a state of pure conduction, there will be no fluid motion and hence we take $\psi_0 = 0$. As the temperature distribution is centrally symmetrical, the function T_0 is found from the solution of the equation

$$\frac{\partial T_0}{\partial t} = \frac{1}{R^2}\frac{\partial}{\partial R}\left(R^2\frac{\partial T_0}{\partial R}\right). \quad (16)$$

Setting $T_0(R, t) = F(R, t)/R$, we get from equation (16)

$$\partial F/\partial t = \partial^2 F/\partial R^2 \quad (16a)$$

and by applying the Laplace transform, we finally obtain,

$$T_0(R, t) = (R_0^2/R)\{\operatorname{erfc}[(R - R_0)/2\sqrt{t}] - \exp(R/R_0 - 1 + t/R_0^2)\operatorname{erfc}[(R - R_0)/2\sqrt{t} + \sqrt{t}/R_0]\}. \quad (17)$$

This solution does not depend on R and t individually, but through a single non-dimensional variable $R/2\sqrt{t}$. Setting $\eta = R/2\sqrt{t}$ and $\eta_0 = R_0/2\sqrt{t}$, we obtain from equation (17),

$$T_0(\eta) = R_0(\eta_0/\eta)[\operatorname{erfc}(\eta - \eta_0) - \exp(\eta/\eta_0 - 1 + 1/4\eta_0^2)\cdot\operatorname{erfc}(\eta - \eta_0 + 1/2\eta_0)]. \quad (17a)$$

The first convective correction to the velocity field is now found from the solution of the equation

$$\frac{1}{R^2}\frac{\partial}{\partial\phi}\left(\frac{1}{\sin\phi}\frac{\partial\psi_1}{\partial\phi}\right) + \frac{1}{\sin\phi}\frac{\partial^2\psi_1}{\partial\phi^2} = R\sin\phi\cdot\frac{\partial T_0}{\partial R} \quad (18)$$

in which the variables are separated by setting

$$\psi_1 = 2\sqrt{t}R_0^2\sin^2\phi\cdot f(\eta). \quad (19)$$

This then leads to an ordinary differential equation of the second order for $f(\eta)$:

$$\eta^2 f''(\eta) - 2f(\eta) = -\eta\operatorname{erfc}(\eta - \eta_0) + \eta(1 - \eta/\eta_0)\cdot\exp(\eta/\eta_0 - 1 + 1/4\eta_0^2)\cdot\operatorname{erfc}(\eta - \eta_0 + 1/2\eta_0), \quad (20)$$

where the primes denote differentiation with respect to η. The general solution of this equation is found to be

$$f(\eta) = C_1\eta^2 + C_2/\eta + \eta_0(\eta_0/\eta - 1)\cdot\exp(\eta/\eta_0 - 1 + 1/4\eta_0^2)$$

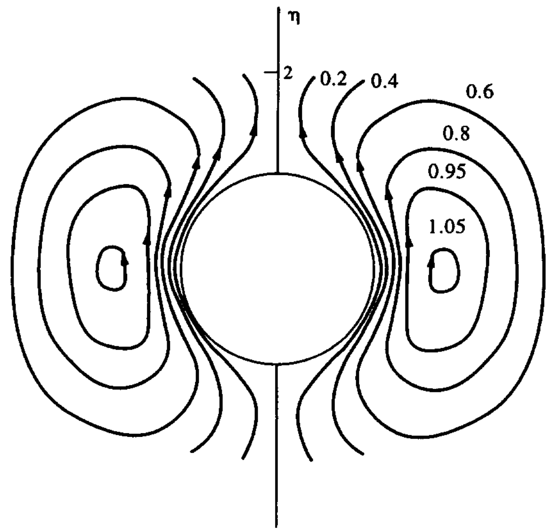

Fig. 2. Transient flow pattern. Curves represent the streamlines $(\psi_1/2R_0^2)t^{-1/2}$ = const. with $\eta_0 = 1$.

$$
\begin{aligned}
&\cdot \operatorname{erfc}(\eta-\eta_0+1/2\eta_0)+f_1(\eta)\\
&\cdot \operatorname{erfc}(\eta-\eta_0)+f_2(\eta)\\
&\cdot \exp[-(\eta-\eta_0)^2]+f_3(\eta)\\
&\cdot \int_{\eta_0}^{\eta} \exp[-(\eta-\eta_0)^2]/\eta\cdot \mathrm{d}\eta
\end{aligned} \tag{21}
$$

where

$$
\begin{aligned}
f_1(\eta) &= \eta^2(\eta_0^3/3-\eta_0/2+\sqrt{\pi}/2)-(2\eta_0^2+1)/4\eta\\
f_2(\eta) &= (1/\sqrt{\pi})[\eta(2\eta_0^3/3+\eta_0+\sqrt{\pi})\\
&\qquad +\eta_0(1-\eta\eta_0)/2\eta+1/4]\\
f_3(\eta) &= \eta^2(-10\eta_0^4/3+4\eta_0^3-7\eta_0^2/3\\
&\qquad +\eta_0\sqrt{\pi}-1/2)/\sqrt{\pi}.
\end{aligned} \tag{22a}
$$

The boundary conditions at infinity imply

$$
C_1 = (1/\sqrt{\pi})(10\eta_0^4/3-4\eta_0^3+7\eta_0^2/3 -\eta_0\sqrt{\pi}+1/2)\cdot X(\eta_0) \tag{22b}
$$

where

$$
X(\eta_0) = \exp(-\eta_0^2)\cdot\left[\sqrt{\pi}\int_0^{\eta_0} \mathrm{e}^{\varepsilon^2}\,\mathrm{d}\varepsilon-\frac{1}{2}\mathrm{e}_i(\eta_0^2)\right] \tag{22c}
$$

$\mathrm{e}_i(\cdot)$ being the exponential integral. The boundary condition on the surface of the sphere implies

$$
\begin{aligned}
C_2 = {} & -C_1\eta_0^3-\eta_0^3(\eta_0^2/3-\eta_0/2+\sqrt{\pi}/2)\\
&+(2\eta_0^2+1)/4-(1/\sqrt{\pi})[\eta_0(2\eta_0^3/3\\
&+\eta_0+\sqrt{\pi})-(2\eta_0^2-1)/4+1/2].
\end{aligned} \tag{22d}
$$

The map of the streamlines $\psi_1/(2\sqrt{t}R_0^2)$ = const. is presented in Fig. 2, from which it is deducible that in the initial stages, closed loops appear in the flow whose geometry considerably changes at higher values of t, although the symmetry about the plane $\phi = \pi/2$ is preserved at all times. Furthermore, during the initial stages of the flow development, there is a concentration of the velocity field around the sphere and as time increases the flow pattern present near the sphere spreads outwards filling the entire space. These streamlines enclose the stagnation points (points of maximum ψ) whose distance from the centre of the sphere increases with time and finally recede to infinity in the ultimate state as $t \to \infty$. There is no accumulation of heat into the flow field and the flow in and around the streamlines remains laminar, the entire process being dominated by viscosity coupled with thermal diffusion. The smaller the radius of the sphere, the lesser will be the concavity of the streamlines in its vicinity and in the limiting case when $\eta_0 \to 0$, the streamlines instead of being concave will tend to be convex, so that there will be a slight bulging of the streamlines in the neighbourhood of the origin. As this situation corresponds to that of a point heat source of thermal energy Q embedded in an unbounded medium, it is natural that we recover the results of Bejan [5].

The first convective correction to the thermal field is now found from the solution of the equation,

$$
\frac{\partial T_1}{\partial t}+\frac{1}{R^2\sin\phi}\left[\frac{\partial(\psi_0,T_1)}{\partial(\phi,R)}+\frac{\partial(\psi_1,T_0)}{\partial(\phi,R)}\right]
=\frac{1}{R^2}\left[\frac{\partial}{\partial R}\left(R^2\frac{\partial T_1}{\partial R}\right)+\frac{1}{\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial T_1}{\partial\phi}\right)\right] \tag{23}
$$

in which the separation of variables can be achieved by setting

$$
T_1 = \cos\phi\cdot G(\eta)/\sqrt{t}. \tag{24}
$$

This then leads to a second order differential equation for G:

$$
\begin{aligned}
&\eta^2G''(\eta)+2\eta(\eta^2+1)G'(\eta)+2(\eta^2-1)G(\eta)\\
&\quad = -[R_0^2/\eta]^2\cdot[\operatorname{erfc}(\eta-\eta_0)-(1-\eta/\eta_0)\\
&\quad \cdot\exp(\eta/\eta_0-1+1/4\eta_0^2)\\
&\quad \cdot\operatorname{erfc}(\eta-\eta_0+1/2\eta_0)]\cdot f(\eta),
\end{aligned} \tag{25}
$$

with the boundary conditions $G'(\eta_0) = G(\infty) = 0$. As an exact solution is not possible, for the sake of completeness we solve equation (25) numerically choosing $\eta_0 = 1$ and plot the graph of $G(\eta)$ in Fig. 3, from which it is deducible that $G(\eta)$ increases from zero on the surface of the sphere to a certain value and then starts decreasing to zero at positions farther away from the sphere. Albeit tacitly, this then implies that there is an increase in temperature for points in the upper half-space $(0 \leqslant \phi < \pi/2)$ accompanied by an equal decrease in temperature in the lower half-space, and consequently the fluid particles in the upper half-space will be warmer than those in the lower half-space.

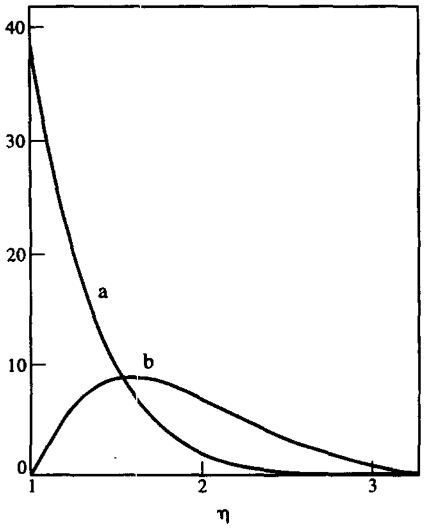

Fig. 3. Temperature profiles of the transient state: (a) T_0/R_0; (b) $(\sqrt{t}/\cos\phi)T_1$ with $\eta_0 = 1$.

However, in the limiting case when the heated sphere reduces to a point heat source, the behaviour of the function $G(\eta)$ will be otherwise, since, by virtue of being a point source, it is a point of singularity [5] and consequently the velocity as well as the temperature is infinite at the origin. One could further observe from the figure that the contribution of the conduction solution to the heat transfer mechanism is more pronounced than the first-order convective correction to the temperature field, especially in the immediate neighbourhood of the sphere which is the important region of thermal activity.

Owing to prohibitive algebraic complexities, higher-order corrections to the velocity and temperature fields were not obtained.

4. STEADY STATE

In the ultimate state (as $t \to \infty$), the equations governing the flow and temperature in the non-dimensional form reduce to

$$\frac{1}{R^2}\frac{\partial}{\partial\phi}\left(\frac{1}{\sin\phi}\frac{\partial\psi}{\partial\phi}\right)+\frac{1}{\sin\phi}\frac{\partial^2\psi}{\partial R^2} = Ra\left(\cos\phi\frac{\partial T}{\partial\phi}+R\sin\phi\frac{\partial T}{\partial R}\right) \quad (26)$$

$$\frac{1}{\sin\phi}\frac{\partial(\psi,T)}{\partial(\phi,R)} = \frac{\partial}{\partial R}\left(R^2\frac{\partial T}{\partial R}\right)+\frac{1}{\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial T}{\partial\phi}\right) \quad (27)$$

with the associated boundary conditions

$$\begin{aligned} &U, V, T \to 0 \quad \text{as } R \to \infty, \\ &(R^2\sin\phi)^{-1}\partial\psi/\partial\phi = 0; \quad \partial T/\partial R = -1 \text{ on } R = R_0 \\ &\partial U/\partial\phi = V = \partial T/\partial\phi = 0 \quad \text{at } \phi = 0, \pi. \end{aligned} \quad (28)$$

Assuming the series expansions as in equation (15) we have for the steady state $\psi_0 = 0$ and

$$T_0 = R_0^2/R \quad (29)$$

which is otherwise evident from equation (17) in the limit $t \to \infty$. For the sake of brevity we present below the solutions of the non-vanishing coefficients of equation (15) up to the order of $(Ra)^2$:

$$\begin{aligned} \psi_1 &= (R_0^3/2)(\eta-1/\eta)\sin^2\phi \\ T_1 &= (R_0^3/2)[1/\eta-(5/4)\eta^{-2}+(1/2)\eta^{-3}]\cos\phi \\ \psi_2 &= (R_0^5/4)[2\eta/3-5/4+1/\eta \\ &\quad -(5/12)\eta^{-2}]\sin^2\phi\cdot\cos\phi \\ T_2 &= (R_0^5/4)[f_1(\eta)+f_2(\eta)\cos^2\phi] \end{aligned} \quad (30)$$

where,

$$\begin{aligned} f_1(\eta) &= -(17/18)\eta^{-1}+(5/4)\eta^{-2}-(2117/3780)\eta^{-3} \\ &\quad -(11/30)\eta^{-3}\log\eta-(5/72)\eta^{-4}+(1/140)\eta^{-5} \\ f_2(\eta) &= (5/6)\eta^{-1}-(5/2)\eta^{-2}+(2537/1260)\eta^{-3} \\ &\quad +(11/10)\eta^{-3}\log\eta-(5/12)\eta^{-4}+(5/28)\eta^{-5} \end{aligned} \quad (30a)$$

and $\eta = R/R_0$.

In order to illustrate the solution, we have drawn in Fig. 4 the streamlines and the isothermal lines corresponding to two different values of Ra, with the left-half of the map showing the isothermal lines and the right-half the streamlines of the motion. While the streamlines are a little wider apart below the sphere than above, the isothermal lines are not concentric owing to convection and in the second case ($Ra = 6$) the maps of those isothermal lines which are a little farther away from the sphere do not exhibit closed loops. It is further observed from the figures that the streamlines are unicellular when $Ra = 1$ whereas they are multi-cellular when $Ra = 6$. In fact, with

$$\psi = Ra\psi_1+Ra^2\psi_2, \quad (31)$$

it is seen that for the dividing streamline $\psi = 0$,

$$R = R_0; \quad \phi = 0, \pi \quad (31a)$$

and the (R, ϕ) points satisfy the relation

$$\cos\phi = -\frac{24}{Ra}\cdot\frac{R(R+R_0)}{(8R^2-7RR_0+5R_0^2)}. \quad (31b)$$

This then leads to a range of values of Ra such that when $0 < Ra \leqslant 3$, the flow pattern is unicellular, and parts of the streamlines below the sphere move off the symmetry axis whereas, when $Ra > 3$, in the relatively colder region below the surface of the sphere a second cell appears. Appropriate numerical values of $\cos\phi$ from equation (31b) suggest that the second cell appears below the sphere at a radial distance of about $(13/2)R_0$ from the centre of the sphere, when $Ra = 4$

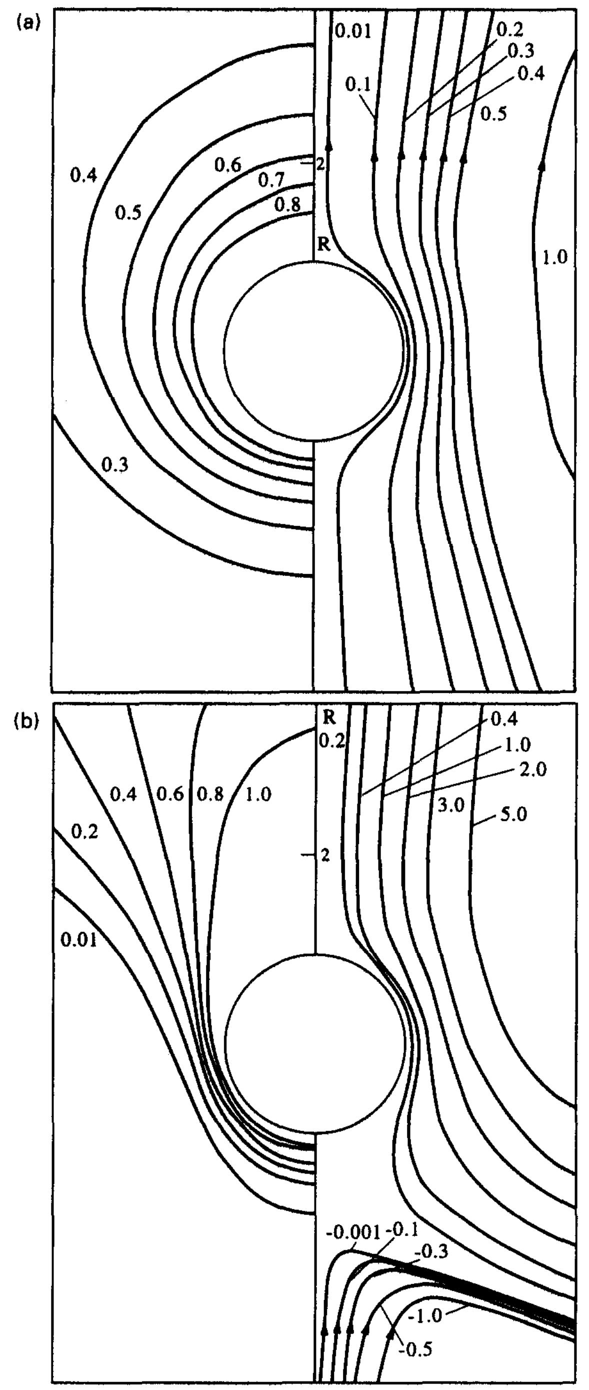

Fig. 4. Profiles of streamlines (right half) and isothermal lines (left half): (a) $Ra = 1$ (b) $Ra = 6$ with $R_0 = 1$.

and at a radial distance of about $(7/2)R_0$, when $Ra = 5$ etc.

The total quantity of heat necessary to maintain the steady-state is characterised by the local Nusselt number defined by

$$Nu = \int_s T_{(R=1)}\,ds \Big/ \int_s T_{0_{(R=1)}}\,ds, \tag{32}$$

where s is the surface of the unit sphere. Using the expressions for T and T_0, we find from equation (32)

$$Nu = 1-(0.07025)Ra^2+O(Ra^3), \tag{33}$$

which implies that the effect of convection on heat transfer is of the order $O(Ra^2)$, which is for small values of Ra. This explains the reason why porous media are widely used in thermal insulation of heated bodies.

5. CONCLUSION

We have presented a theoretical study of natural convection from a sphere of constant heat flux buried instantaneously in an unbounded fluid-saturated porous medium and obtained the transient and steady-state solutions for the flow and temperature fields using a perturbation analysis in the limit of small Rayleigh numbers. The results are valid in the diffusion dominated regime only. As the thermal Rayleigh number is assumed to be small, the solutions we have obtained for the streamfunction are expected to give a reasonably good picture of the free convection motion and in the absence of stability effects, the behaviour of the flow is also unlikely to change radically for moderate values of Ra. Since the Rayleigh number depends only on the total heat flux from the sphere and the permeability of the medium, the evolution of the flow pattern is expected to be the same, irrespective of the nature of the material embedded. For instance, when a copper sphere is embedded in a surrounding sand–oil medium, depending on the properties of the saturating fluid and the total heat flux from the sphere one can expect the Rayleigh number to be as high as of $O(1)$, since the permeability of a sand–oil medium varies from 2×10^{-7} to 1.8×10^{-6} [6].

Of special importance is the finding that for $Ra > 3$, there appears a second cell below the sphere, so that the flow pattern is multi-cellular and in this case, the isothermal lines that are formed a little farther away from the sphere do not exhibit closed loops. Our results show that the convective flow near the top of the sphere retains its axial symmetry and do not show any evidence of a third cell forming there, breaking the symmetry between the conditions at the sphere top and bottom.

Acknowledgement—The author is thankful to the referees for their many useful suggestions which led to a definite improvement of the paper.

REFERENCES

1. Yamamoto, K., Natural Convection about a Heated Sphere in a Porous Medium. *Journal of the Physical Society of Japan*, 1974, **37**, 1164–1166.
2. Lai, F. C. and Kulacki, F. A., Coupled heat and mass transfer from a sphere buried in an infinite porous medium. *International Journal of Heat and Mass Transfer*, 1990, **33**, 209–215.
3. Ganapathy, R. and Purushothaman, R., Free convection

in an infinite porous medium induced by a heated sphere. *International Journal of Engineering Science*, 1990, **28**, 751–759.
4. Cheng, P., Heat transfer in geothermal systems. In *Advances in Heat Transfer*, Vol. 14. Academic Press, New York 1978, pp. 1–105.
5. Bejan, A., Natural convection in an infinite porous medium with a concentrated heat source. *Journal of Fluid Mechanics*, 1978, **89**, 97–107.
6. Scheideggar, A. E., *The Physics of Flow through Porous Media*. University of Toronto Press, Toronto, ON 1963, p. 87.